IP-PCG

An interior point algorithm for nonlinear constrained optimization

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Overview

IP-PCG is a C++ software designed to solve large-scale Nonlinear Programming Problems (NLP) with equality, inequality and box constraints. It employs a Newton inexact interior point algorithm [1, 2, 3, 4] with a line search strategy based on the Eisenstat and Walker rule, even in the nonmonotone case [5]. At each step of the interior point algorithm, a perturbation of the Newton equation is solved by the Preconditioned Conjugate Gradient (PCG) method, with a suitable indefinite preconditioner [6, 7]. The main features of the code are:

- Sparse matrix representation;
- Iterative solution of the inner linear system with an adaptive stopping criterion;
- Four different representation of the Newton system;
- Direct factorization of the preconditioner;
- Three different PCG algorithms;
- Partial AMPL interface;
-

Algorithm

The interior point algorithm implemented by IP-PCG is derived from the algorithm presented in [1]. The main difference consists in the choice of the perturbation parameter which allows to develop the global convergence

theory in the framework of the inexact Newton methods [2, 3, 4]. Furthermore, this allows an approximate solution of the Newton system which has to be solved at each step of the interior point algorithm. The approximate solution of the linear system is computed with an adaptive tolerance by the PCG method with a suitable indefinite preconditioner [6, 7].
The line search strategy employs the Eisenstat and Walker acceptance rule, even in the nonmonotone case presented in [5].

The code

A binary version of the code is available at

http://dm.unife.it/pn2o/software

for the HP-Itanium2 platform. The source code is available on demand by the authors.

How to use

IP-PCG should be called by the following command line

./ip_pcg stub.nl [options]

where stub.nl is the AMPL generated .nl file of an NLP problem.
The available keywords are

keyword	arguments	description
alg	[2..5]	representation of the Newton system [7]
		2 = block preconditioner 2X2
		3 = block preconditioner 3X3 (default)
		4 = block preconditioner 4X4
		5 = active-inactive preconditioner (Luksan)
fctroutine	[1,2]	factorization routine selection
		1 = routine BLKFCLT (default)
		2 = MA27 of the Harwell Subroutine Library
ignore_initsol	[0,1]	ignore the initial estimate
		0 = assume the starting point specified in the AMPL model (set to 0 if not specified)
		1 = the algorithm choices the initial point (default)
maxit	*integer*	maximum number of iterations (default=500)
maxtime	*integer*	maximum time in seconds (default=7200)
mon_deg	*integer*	monotonicity degree (see [5])
		1 = monotone algorithm (default)
		>1 = nonmonotone algorithm
mufactor	[1,2]	choice of the perturbation parameter
		1 = central path (default)
		2 = inexact Newton
sol	[0,1]	print of the results
		0 = no print (default)
		1 = print of the solution vector and the multiplier vector on the file risultati.dat
tol	*float*	optimality error tolerance (default = 1e-8)
typepcg	[1..3]	PCG Algorithm selection (see [8])
		1 = Algorithm 2.1
		2 = Algorithm 2.2
		3 = Algorithm 2.3
version		version and last update date

This is an example of the output prints produced by the code on the test problem SVANBERG of the CUTE collection:

```
*****************************************
*                IP-PCG                 *
*****************************************
primal variables= 5000
equality constraints= 0
inequality constraints= 5000
lower bounds= 5000
```

```
 upper bounds= 5000
 Iter |      obj       |    ||KKT||     |    ||step||    | nred | itcg |  ma  |  mi
--------------------------------------------------------------------------------
    0 | 1.374850e+04 | 4.773814e+02 |
    1 | 1.392771e+04 | 3.906088e+02 | 2.718851e+02  |    0 |    1 |  5000 |     0
    2 | 1.360448e+04 | 2.862918e+02 | 2.000735e+02  |    0 |    1 |  5000 |     0
    3 | 1.183760e+04 | 1.456688e+02 | 1.825621e+02  |    0 |    1 |  5000 |     0
    4 | 9.796939e+03 | 6.308225e+01 | 5.922460e+01  |    0 |    1 |  5000 |     0
    5 | 9.040129e+03 | 3.412348e+01 | 9.644112e+01  |    0 |    1 |  5000 |     0
    6 | 8.716743e+03 | 1.833104e+01 | 8.886510e+01  |    0 |    1 |  5000 |     0
    7 | 8.509860e+03 | 1.231068e+01 | 7.867935e+01  |    0 |    1 |  5000 |     0
    8 | 8.416826e+03 | 6.019283e+00 | 5.545170e+01  |    0 |    1 |  5000 |     0
    9 | 8.385515e+03 | 2.281207e+00 | 2.666528e+01  |    0 |    1 |  5000 |     0
   10 | 8.372562e+03 | 9.605530e-01 | 1.294000e+01  |    0 |    1 |  5000 |     0
   11 | 8.365633e+03 | 3.279425e-01 | 6.824475e+00  |    0 |    1 |  5000 |     0
   12 | 8.363134e+03 | 1.317759e-01 | 3.411674e+00  |    0 |    1 |  5000 |     0
   13 | 8.362110e+03 | 5.102537e-02 | 1.771539e+00  |    0 |    1 |  5000 |     0
   14 | 8.361708e+03 | 2.272911e-02 | 9.697211e-01  |    0 |    1 |  5000 |     0
   15 | 8.361543e+03 | 1.052232e-02 | 5.526477e-01  |    0 |    1 |  5000 |     0
   16 | 8.361474e+03 | 4.895011e-03 | 3.212531e-01  |    0 |    1 |  4999 |     1
   17 | 8.361445e+03 | 2.268611e-03 | 1.897909e-01  |    0 |    1 |  4999 |     1
   18 | 8.361433e+03 | 1.042448e-03 | 1.151085e-01  |    0 |    1 |  4998 |     2
   19 | 8.361428e+03 | 4.746402e-04 | 7.595660e-02  |    0 |    1 |  4997 |     3
   20 | 8.361426e+03 | 1.670893e-04 | 6.099527e-02  |    0 |    1 |  4995 |     5
   21 | 8.361425e+03 | 6.657024e-05 | 7.184297e-02  |    0 |    1 |  4456 |   544
   22 | 8.361425e+03 | 2.732431e-05 | 9.786548e-02  |    0 |    1 |  4285 |   715
   23 | 8.361424e+03 | 9.360545e-06 | 1.216735e-01  |    0 |    1 |  4188 |   812
   24 | 8.361424e+03 | 3.671676e-06 | 1.394270e-01  |    0 |    1 |  4102 |   898
   25 | 8.361424e+03 | 1.073486e-06 | 8.666421e-02  |    0 |    1 |  4070 |   930
   26 | 8.361424e+03 | 2.884740e-07 | 3.150712e-02  |    0 |    1 |  4051 |   949
   27 | 8.361424e+03 | 3.962208e-08 | 9.174618e-03  |    0 |    1 |  4043 |   957
   28 | 8.361424e+03 | 3.376122e-09 | 1.266544e-03  |    0 |    1 |  4035 |   965


Object function value          : 8.361424e+03
# of nonlinear iterations      : 28
# of pcg iterations            : 28
# of function evaluations      : 29
# of gradient evaluations      : 29
# of Hessian evaluations       : 28
Final optimality error         : 3.376122e-09
CPU time(sec)                  : 4.870000
```

For each iteration the following information are displayed:

- the value of the objective function (obj);
- the euclidean norm of the violation of the Karush–Kuhn–Tucker optimality conditions (||KKT||);
- the euclidean norm of the step, namely the solution of the Newton equation (||step||);
- the number of backtracking reductions performed (nred);
- the number of PCG iterations (itcg);
- the number of active (ma) and inactive (mi) inequality constraints.

Finally, some summary information are displayed after some stopping criterion is satisfied:

- the total number of interior point iterations;
- the total number of PCG iterations;
- the total number of function, gradient and Hessian evaluations;
- the final optimality error;
- the CPU time in seconds (excluding the interface time).

References

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