

The manuscript of Abel's Parisian memoir found in its entirety

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Abstract. The manuscript of Abel's Parisian memoir on transcendental functions was lost after the printing, and only in 1952 that part of this manuscript was found by V. Brun in Florence. Recently, the eight still missing pages of the manuscript have been also found, thus putting an end to more than a century and a half of investigations.

Sunto. Dopo la stampa, il manoscritto della memoria parigina di Abel sulle funzioni trascendenti andò smarrito, e soltanto nel 1952 parte di esso fu ritrovato da V. Brun a Firenze. Le otto pagine ancora mancanti del manoscritto sono state ritrovate recentemente, ponendo fine a un secolo e mezzo di ricerche.

1991 Mathematical Subject Classification: 01A50, 01A70, 01A90

Key words: Abel, transcendental function.

On October 30, 1826, Niels Henrik Abel presented the Paris Academy of Sciences with a work entitled *Mémoire sur une propriété générale d'une classe très étendue de fonctions transcendentes*. Abel had arrived in Paris on July 10th and after a while he began to working on what was to become his most famous memoir. On October 24 he wrote to his friend Holmboe:

Je viens de finir un grand traité sur une certaine classe de fonctions transcendentes... Et j'ose dire, sans vanter, que c'est un bon travail. Je suis curieux d'entendre l'opinion de l'Institut. [Abel, N. H. 1881, II, 260].

What would it not have meant for his career to return to Norway with his memoir having been praised by the Academy? For the rest of his stay in Paris Abel was just waiting for an answer from the Academy but, as long as he lived, he never got it (see [Ore 1957, 150-151] or the more recent [Stubhaug 2000, 411]). A few months before he died Abel wrote a short account of his results [Abel 1829]

As is well known, Cauchy, who had been named to referee Abel's work together with Legendre, lost the manuscript, and this memoir was not published until 1841 [Abel, N. H. 1841]¹. Guglielmo Libri, professor of mathematics at both the Collège de France and the Sorbonne, had the responsibility of supervising the printing [Abel 1841, 264]. But Abel's manuscript disappeared again a short time after it was printed. While S. Lie and L. Sylow were preparing a new edition of Abel's *Collected Works*, they attempted in vain to get the Parisian manuscript from the Academy, and in fact they wrote:

Il nous a paru très désirable de pouvoir collationner le mémoire imprimé avec l'original, et M. Sophus Lie obtint en 1874 de l'Académie des Sciences de Paris la permission de consulter le

¹ For the relationships between Abel and the Paris Academy of Science see [Taton 1947].

manuscrit d'Abel; mais il fut constaté dans les archives de l'Académie que le manuscrit ne s'y est pas trouvé après l'impression du mémoire [Abel 1881, II, 294].

On October 1952 part of the manuscript of Abel's celebrated memoir was found in Florence by V. Brun and he reported his discovery in three articles: [Brun 1953], the more detailed [Brun 1954] and the shorter [Brun 1955]. In [Brun 1955, 104] we read:

Le P^r Sansone me signala un chapitre d'un livre publié par Giacomo Candido en 1942: "*Sulla mancata pubblicazione della celebre Memoria di Abel*" où il était mentionné qu'il existait à la bibliothèque Moreniana de Florence un manuscrit intitulé: *A. Legendre - Nota autografa, rinvenuta nel Fondo Palagi-Libri, attaccata alla copia, fatta dal Libri, della «Memoria di Abel*²

and moreover:

Aussi, est-ce avec beaucoup d'émotion qu'aidé par M. le P^r Procissi, j'ai ouvert le vieux manuscrit jauni de la bibliothèque Moreniana proche l'église de Lorenzo.

A microfilm of the manuscript was sent to Oslo where an expert stated that the manuscript had been written by Abel himself. From the same note [Brun 1955, 103] we also learn that in 1942 P. Heegaard from Oslo thought to have found the lost manuscript at the Biblioteca Nazionale of Rome, but a later expertise in Oslo showed that that manuscript was only a handwritten copy of the printed article.

The manuscript found by Brun was incomplete, consisting only of the first 16 pages and the last one only, the pages being numbered from 1 to 16 and 61. What Brun held in his hand was just one-third of the complete text.

In [Brun 1955, 105] we also read:

Quelque temps après le P^r Sansone de Florence me signalait qu'un examen minutieux, feuille par feuille, fait par le P^r Procissi à la bibliothèque Moreniana avait conduit au résultat heureux que tout le manuscrit, sauf 8 pages, était retrouvé

A. Procissi did not leave, at least to my knowledge, any reference to this second part of Abel's original manuscript. With closer and patient scrutiny of the documents kept at the Fondo Palagi-Libri I rediscovered this second part³, and by comparing the two I realized that pp. 21-24 and 31-34 were still missing.

After the discovery of most of Abel's manuscript in the Fondo Palagi-Libri of the Biblioteca Moreniana of Florence, it is quite natural to think that Libri had retained the manuscript after the printing. Libri had a great admiration for Abel, and in fact in 1833 he wrote an impassioned biography of him, focusing on Abel's mathematical discoveries [Libri 1833]. This biography was the only one available for many years.

Owing to a change in the political situation in France, in 1848 Libri was charged of having committed a theft in French public libraries. This was an old accusation against him which resurfaced. Consequently, he escaped to London. It is well known that in spite of his hurry, Libri was able to box and ship masses of manuscripts and books (for a detailed biography of Libri see [Maccioni-Monstert 1995]). What was found in Libri's apartment at the Sorbonne was confiscated by the French authorities. All those documents, letters, and manuscripts are now preserved at the National Library of Paris. So it seems clear that Libri had taken Abel's manuscript with him when he escaped to London.

² Fondo Palagi-Libri: filza 436 ins. 2 (1).

³ Fondo Palagi-Libri: filza 437 ins. 10.

In 1868 Libri felt his life was coming to an end and decided to go back to his birthplace Florence. As he intended to continue in doing research, to write and publish, he brought with him what remained of his library and lots of documents, autograph letters, scientific and historic manuscripts some of them dating from his early youth. Libri left London in June 1868 but settled down in Florence only in December, after a long tormented journey due to his poor health.

The executors to this testament are requested to be Mr. Guglielmo Libri, Mr. Tito Del Rosso and Count Giacomo Manzoni. The latter is especially asked to advise the other two in order that the books and the manuscript sheets which constitute the larger part of my small hineritance will fetch the best price possible.⁴ [Candido 1942, 842]

It is a question of the manuscript which had been left to my responsibility but which are now of mine on the condition that by selling them Libri has the benefit of the higher price.⁵

A great quantity of Libri's documents were sold, at a low price, against the provisions of his will, by his namesake cousin to cover the burial expenses (see [Candido 1942, p. 842]). Part of them were luckily bought by Giuseppe Palagi and later sold to the Province of Florence in 1872. Presently, all this material constitutes the *Fondo Palagi-Libri* of the Biblioteca Morenia (see [Candido 1942, 845-860] or the more recent [Grattan-Guinness 1984]). As already said, it is in this deposition that Abel's manuscript was found, except for the missing eight pages.

The remaining part of Libri's inheritance was preserved by Manzoni himself. When the latter died, his library was sold by auction in Rome but in the fourth volume of the sale

⁵ In italian: "...trattasi dei manoscritti che allora erano affidati alla mia responsabilità, e che adesso sono miei alla condizione che, vendendosi, il Libri risenta l'utile del prezzo maggiore...". Biblioteca Trisi di Lugo (Ravenna, Italy), Fondo Gherardi, lettere di G. Manzoni a S. Gherardi.

catalogue, the one of the manuscripts, only one manuscript which previously belong to Libri is listed [Tenneroni 1894].

In 1959 the Moreniana Library was enriched with a new deposition, constituted of letters, booklets, scientific manuscripts and other documents of about 20.000 sheets. A first description of this was done by Arrighi [Arrighi 1983] and, for its content, it has been named *Nuovo Fondo Libri*. This stock, acquired by the Province of Florence from a Florentine antiquarian, belonged with all probability to Manzoni. This fact is evidenced by the discovery, among the documents of the deposition of a sheet headed *Municipio di Lugo* (i.e. Manzoni's birthplace). This sheet, which concerns the meeting of the town council of January 14, 1879, has the following handwritten note on the back:

Manuscripts of Prof. Guglielmo Libri. They are mainly mathematical in character. To put them in order would require the time that I do not have and I will never have.⁶

This note, written by Count Manzoni after his sale to Boncompagni in 1876, proves that he gave little importance to this part of Libri's documents.

Hence, I think that the *Nuovo Fondo Libri* is composed of the remaining part of the great quantity of letters, autographs, manuscripts that Libri left to Manzoni, after all the sales and mutilations suffered during 90 years. I do not know how this material arrived in the hands of the antiquarian.

While searching for the still missing pages of Abel's manuscript, I had hoped to discover them in this deposition. A meticulous inspection, sheet by sheet, of the content of certain boxes constituting the *Nuovo Fondo Libri*, chosen on the basis of the temporary catalogue, has been fruitful: on July 6, 2000 with great astonishment and much emotion, I found the eight missing pages written by the hand of Abel, so ending more than a century and a half of investigations.

These pages are well preserved, much better than those found by Brun, and perfectly readable.

By comparing these eight pages of the manuscript with the printed articles which appeared in *Mémoires présentés par divers savants* and in the second edition of the *Oeuvres complètes*, I found about 22 linguistic or notational discordances. It seems to me that they are all of minor importance, for instance that of Section 6 where in the printed articles the reference is to formula 14 instead of referring to formula 41 as in the manuscript, and the formula numbered 70 as well as the important formula numbered 103 in print are actually not numbered in the manuscript. Finally I would like to mention some misprints and trivial mistakes corrected by Libri during the printing, such the one on page 23 of the manuscript, where Abel forgot to write the formula numbered 71 (72 in the articles), or that on page 33, where we read " $5m+3=m+3$, d'où $m=1/2$ ", which may suggest that the memoir was written in some hurry.

From these facts, this last finding has mainly a historic interest, and for Abel's second jubilee one can claim that *fortunately, in spite of everything it has gone through, the manuscript of the memoir, to which Abel had attached so much hope has been entirely brought to light!*

Acknowledgement.

⁶ In italian: "Manoscritti del Prof. Guglielmo Libri. Sono per lo più matematici. Per ordinarli occorre il tempo e l'ozio che non ho e che non avrò mai ". *Nuovo Fondo Libri*, cassetta 22, ins. 597, c. 4.

It is a pleasure to thank the librarians of the Biblioteca Moreniana of Florence for their kind assistance during my pursuit of the Fondo Palagi- Libri and Nuovo Fondo Libri, and in particular V. Fraticelli, in charge of the Library, for permission to reproduce some pages of the manuscript.

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